Difference Equations

- Part 2 -

Cobweb diagrams

("Spinnweben"-Diagramme)

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Generating the basic data list

The following program generates a list subsequently to be processed for graphical representation.

```
AnnuityList[y0_, A_, B_, k_] :=
 Module[{i = 0, y = y0, AL = {}},
  AL = Append[AL, {i, y}];
  While[i < k,
   i = i + 1; y = A * y + B; AL = Append[AL, {i, y}]];
  Return[AL] ]

AnnuityList[100000, 1.05, -10000, 15]
```

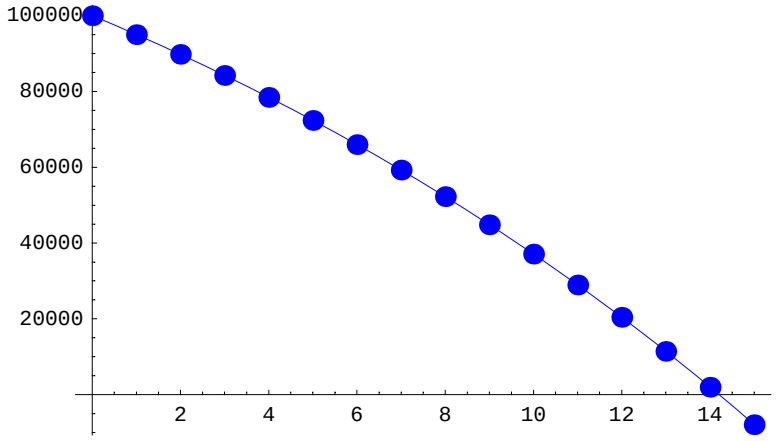
```
{{0, 100000}, {1, 95000.}, {2, 89750.}, {3, 84237.5}, {4, 78449.4}, {5, 72371.8},
 {6, 65990.4}, {7, 59290.}, {8, 52254.5}, {9, 44867.2}, {10, 37110.5},
 {11, 28966.1}, {12, 20414.4}, {13, 11435.1}, {14, 2006.84}, {15, -7892.82}}
```

The standard plot ("timeline plot")

■ Implementation of the timeline plot

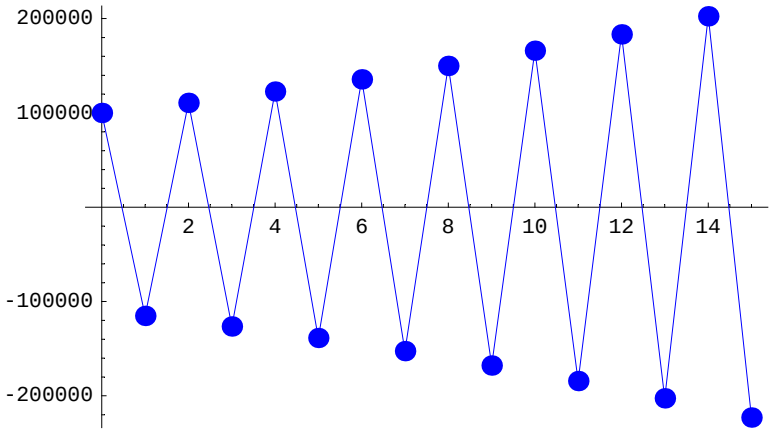
■ Experiments and Results

```
TimelinePlot[100000, 1.05, -10000, 15]
```




```
- Graphics -
```

```
TimelinePlot[100000, -1.05, -10000, 15]
```




```
- Graphics -
```

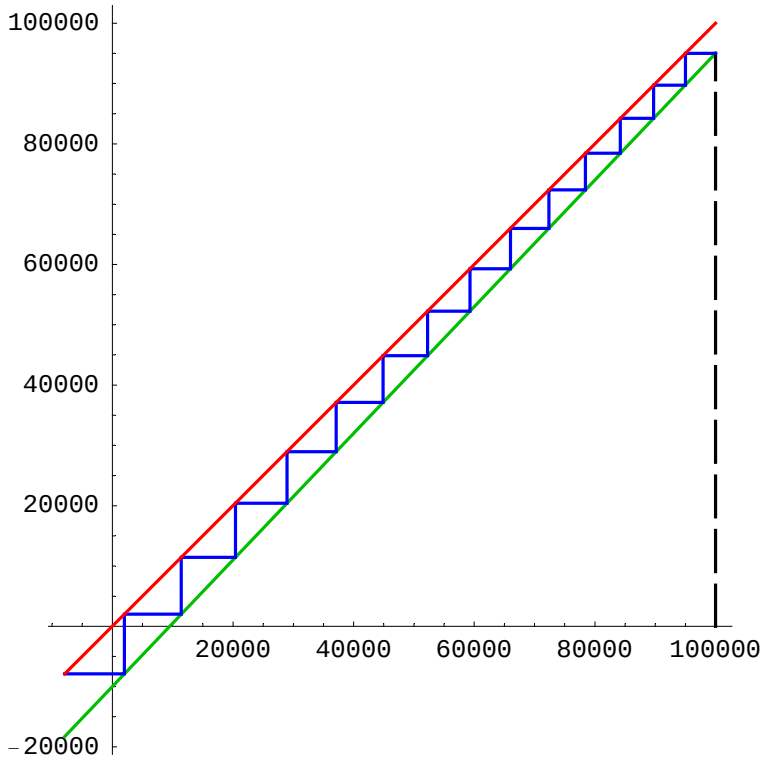
The "cobweb" plot

The program AnnuityList generates a list of pairs, the annuity list. The first component of each pair can be interpetreted as "time". The complete liste gives the development of the magnitude y in time (see previous graphic).

■ Implementation of the cobweb plot

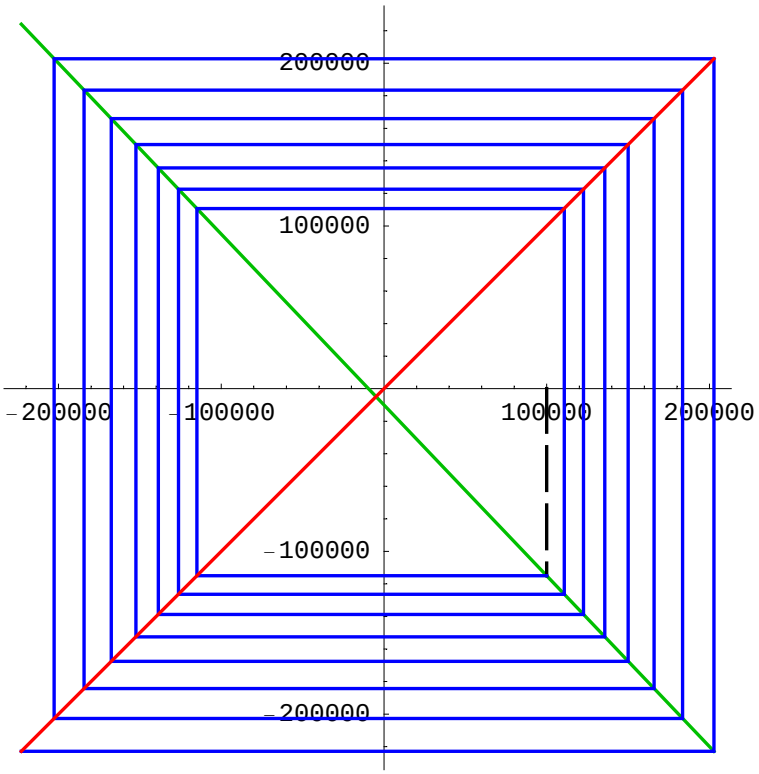
■ Experiments and Results

```
CobWebPlot[100000, 1.05, -10000, 15]
```



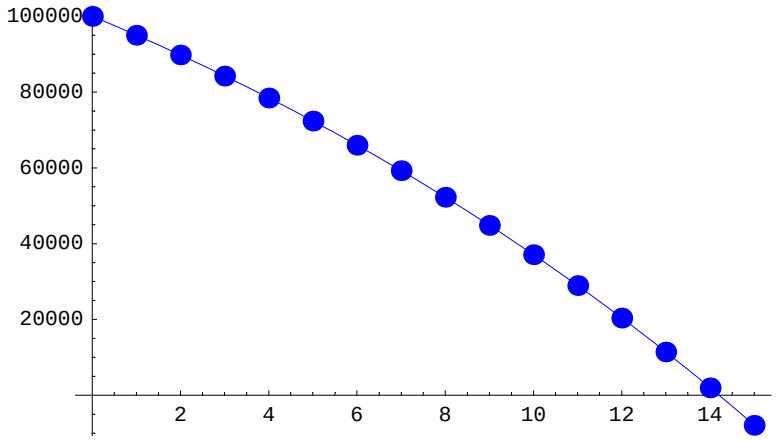

```
- Graphics -
```

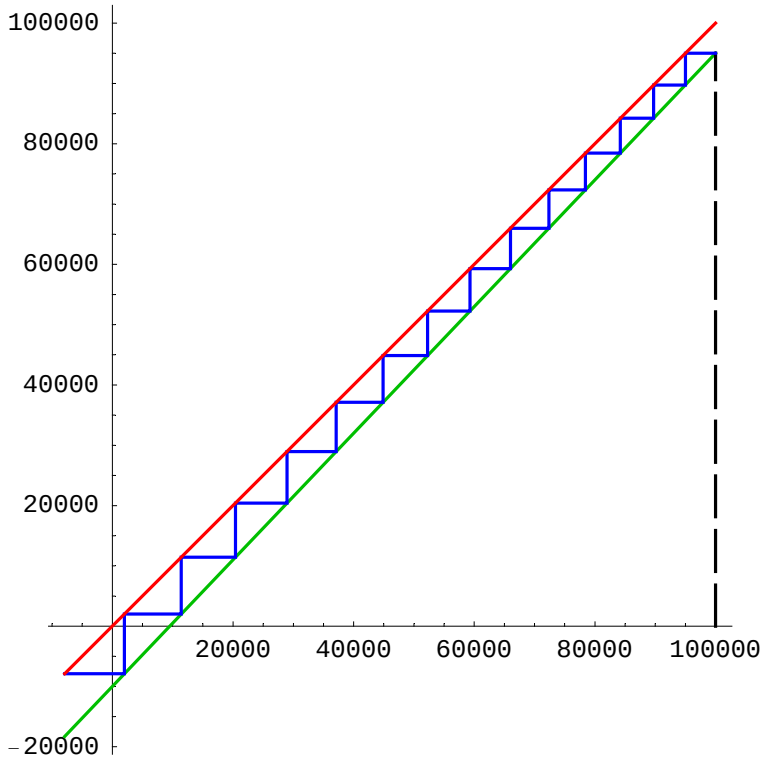
```
CobWebPlot[100000, -1.05, -10000, 15]
```

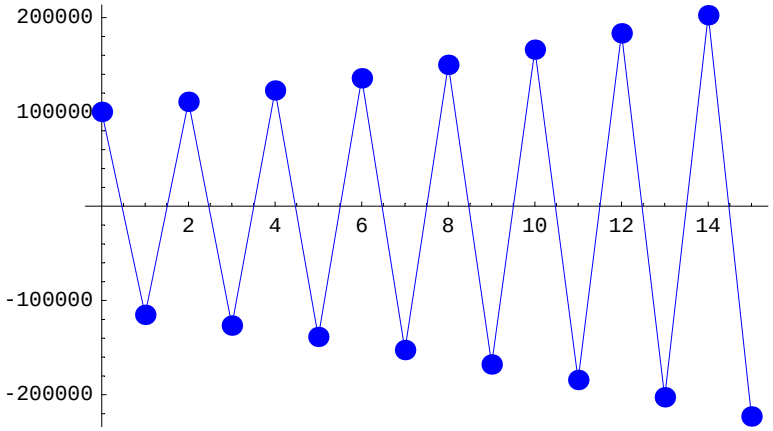


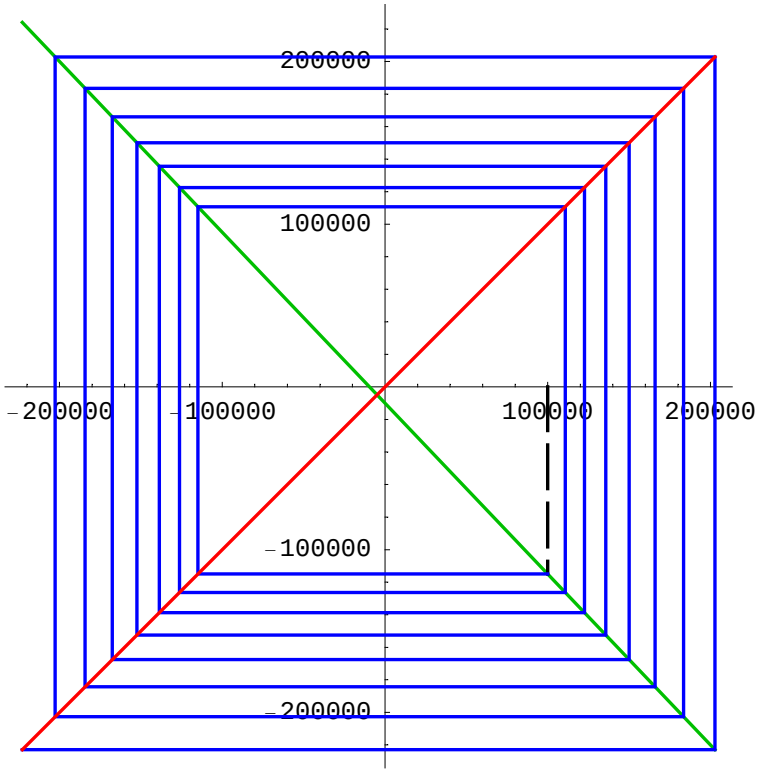

```
- Graphics -
```

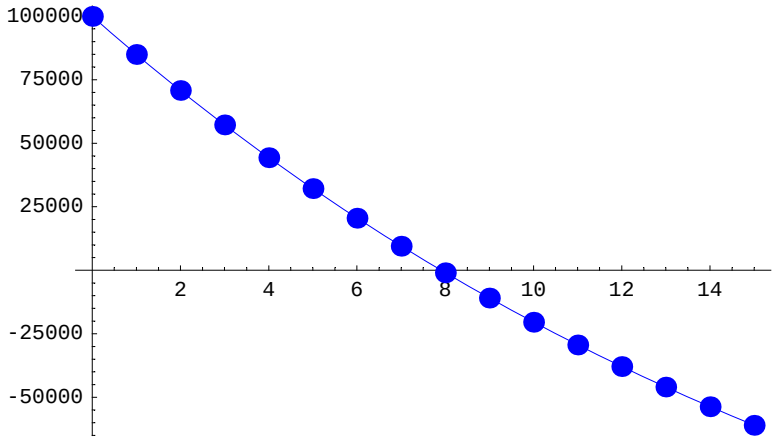
```
Demos =
 (TimelinePlot[100000, 1.05, -10000, 15];
  CobWebPlot[100000, 1.05, -10000, 15];
  TimelinePlot[100000, -1.05, -10000, 15];
  CobWebPlot[100000, -1.05, -10000, 15];
  TimelinePlot[100000, 0.95, -10000, 15];
  CobWebPlot[100000, 0.95, -10000, 15];
  TimelinePlot[100000, -0.95, -10000, 15];
  CobWebPlot[100000, -0.95, -10000, 15] )
```

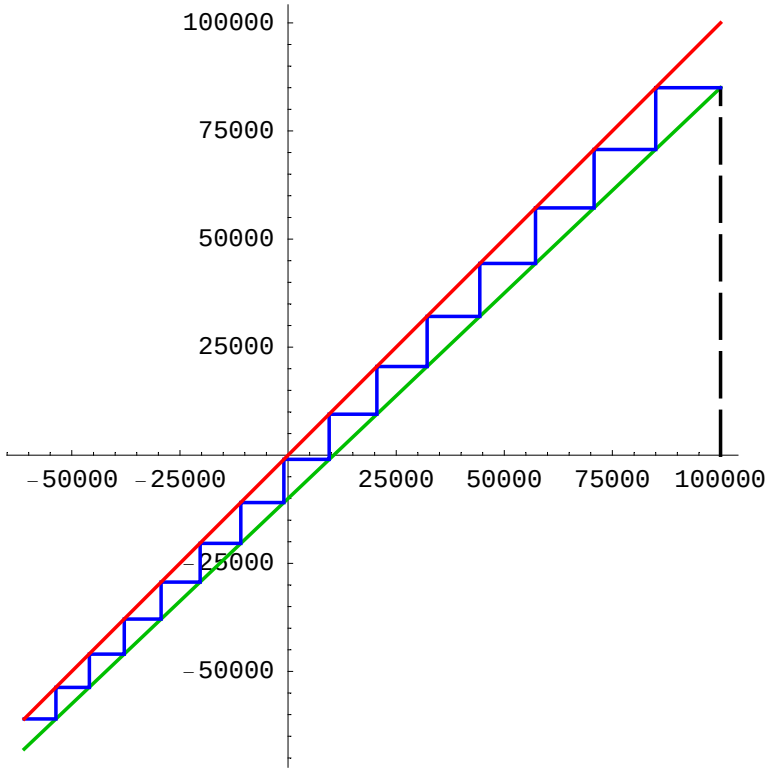

100000
80000
60000
40000
20000
2
4
6
8
10
12
14

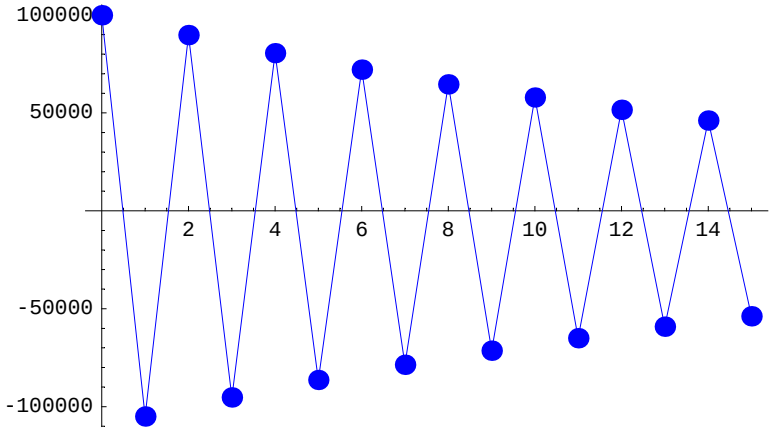

100000
80000
60000
40000
20000
20000
40000
60000
80000
100000
−20000

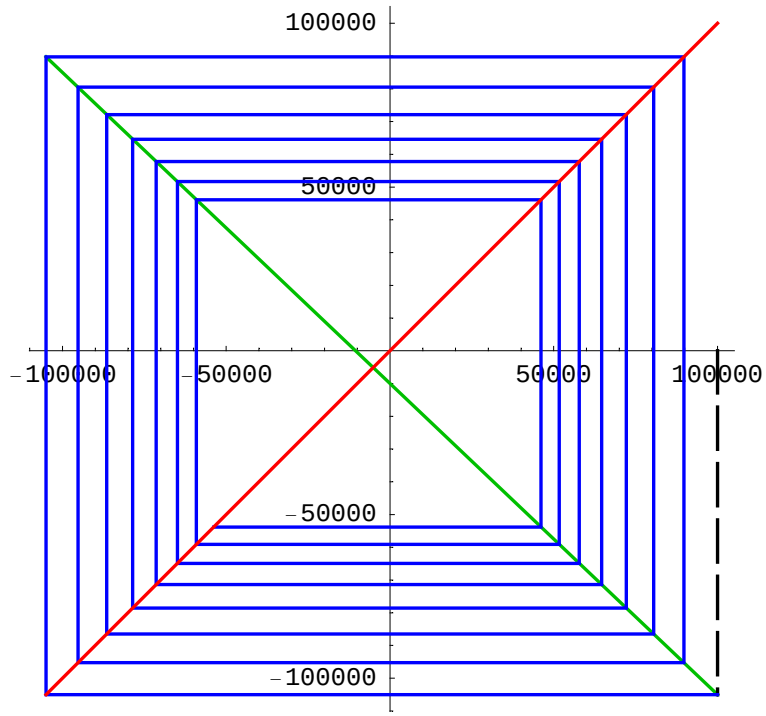

200000
100000
2
4
6
8
10
12
14
-100000
-200000


200000
100000
−200000
−100000
100000
200000
−100000
−200000


100000
75000
50000
25000
-25000
-50000
2
4
6
8
10
12
14


100000
75000
50000
25000
-50000
-25000
25000
50000
75000
100000
-25000
-50000


100000
50000
2
4
6
8
10
12
14
-50000
-100000


100000
50000
-100000
-50000
50000
100000
-50000
-100000

- Graphics -

■ The cobweb plot - some special cases

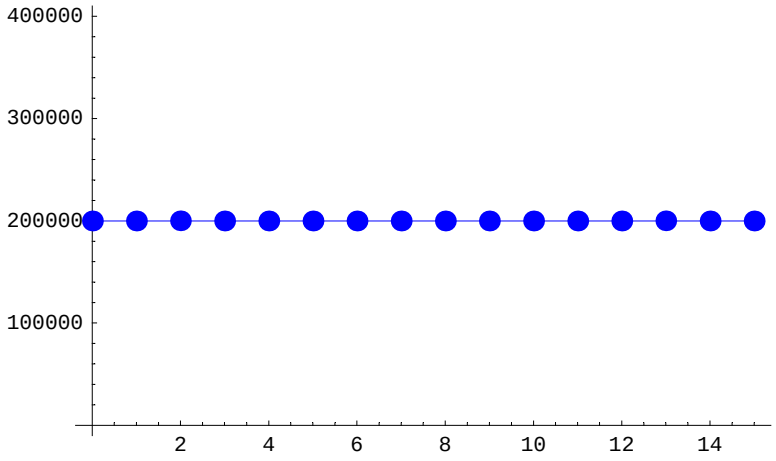
Case: $y_0 = \frac{B}{1-A}$

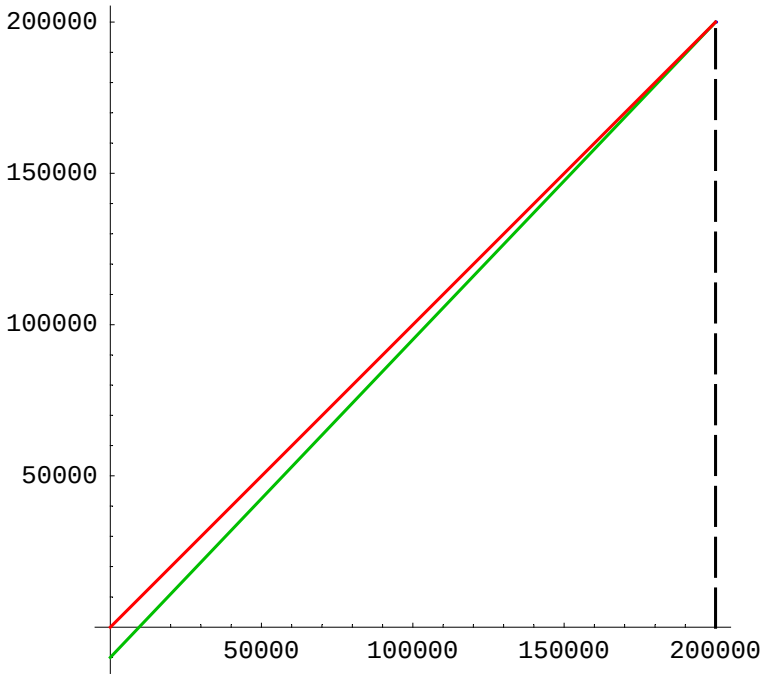
hence: $y_1 = A \cdot y_0 + B = A \cdot \frac{B}{1-A} + B = \frac{A \cdot B + B \cdot (1-A)}{1-A} = \frac{B}{1-A} = y_0$

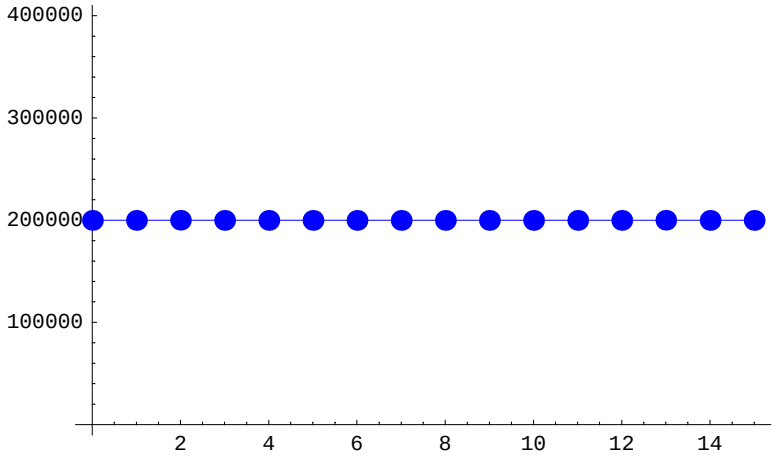
hence: $y_0 = y_1 = y_2 = y_3 = y_4 = \ldots$

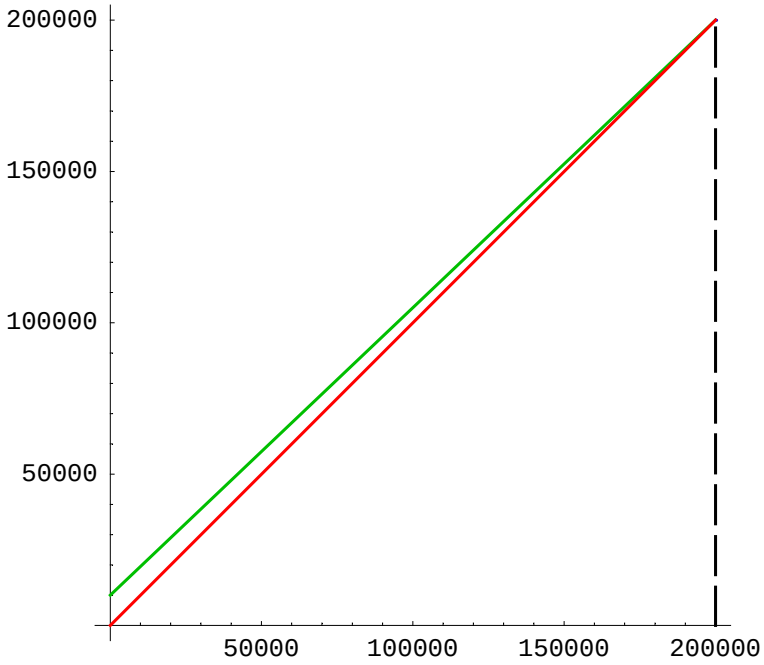
In the following diagram this fact is visualized with various slopes.

```
TimelinePlot[200000, 1.05, -10000, 15];
CobWebPlot[200000, 1.05, -10000, 15];
TimelinePlot[200000, 0.95, 10000, 15];
CobWebPlot[200000, 0.95, 10000, 15];
```




200000
150000
100000
50000
50000
100000
150000
200000


400000
300000
200000
100000
2
4
6
8
10
12
14



Case: $A = -1$

hence: $y_1 = -y_0 + B$

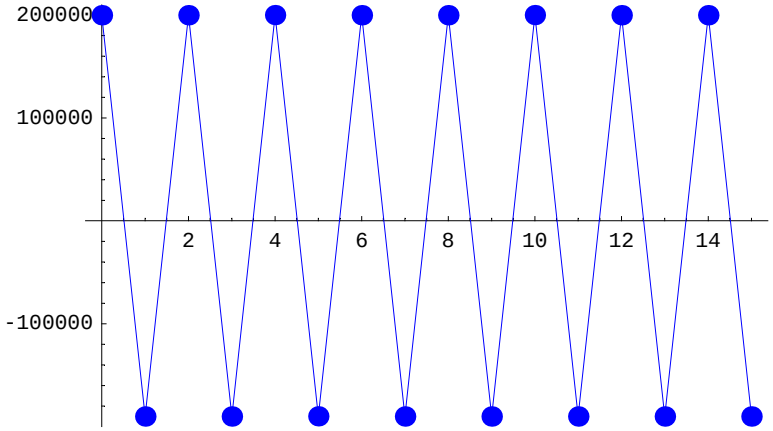
$y_2 = -y_1 + B = (y_0 - B) + B = y_0$

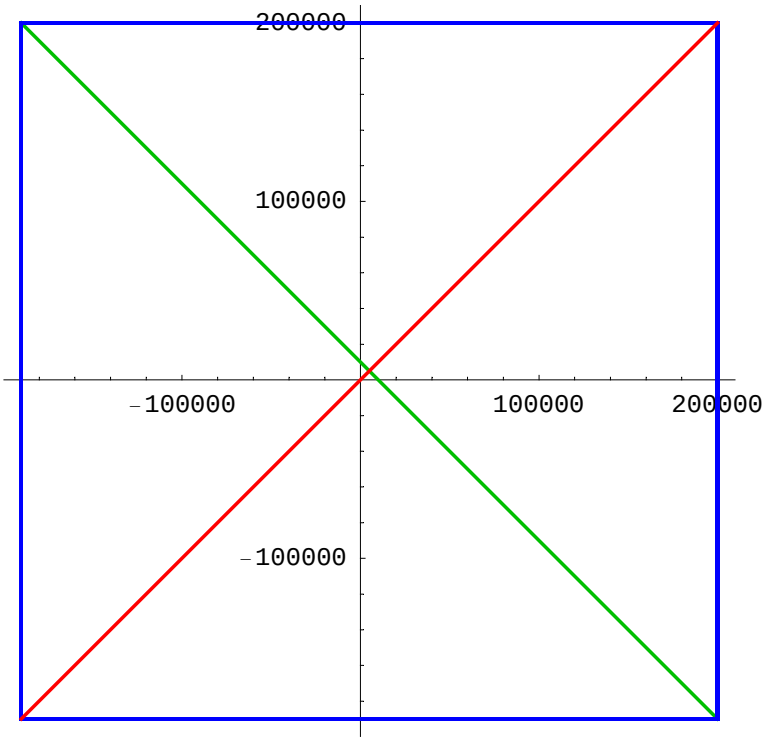
$y_3 = -y_2 + B = -y_0 + B = y_1$

$y_4 = -y_3 + B = -y_1 + B = y_2 = y_0$

Visualisation:

```
TimelinePlot[200000, -1, 10000, 15];
CobWebPlot[200000, -1, 10000, 15];
```





Auxiliary stuff

```
?Global`*
```

```
Remove["Global`*"]
```